

# Optimal monetary policy under menu costs

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February 2024

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Background: **Why does monetary policy matter?**

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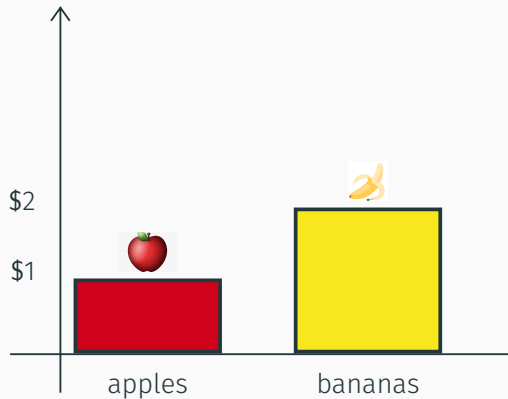
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Benchmark: monetary policy  
*doesn't* matter

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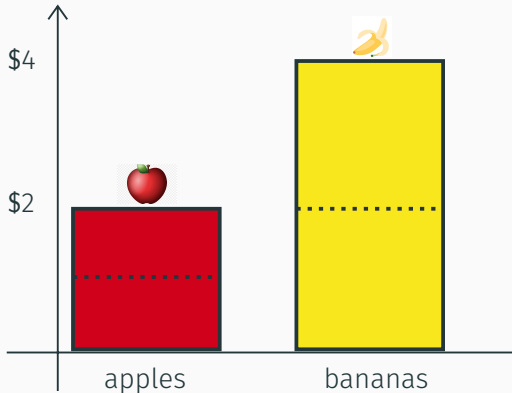


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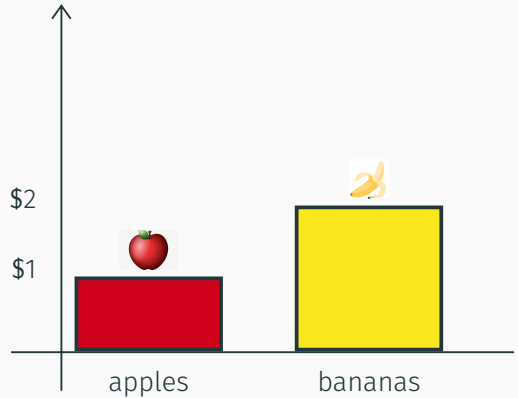
Benchmark: monetary policy *doesn't* matter

- ▶ Money supply doubles  
⇒ all prices double  
⇒ *nothing real affected*  
by monetary policy



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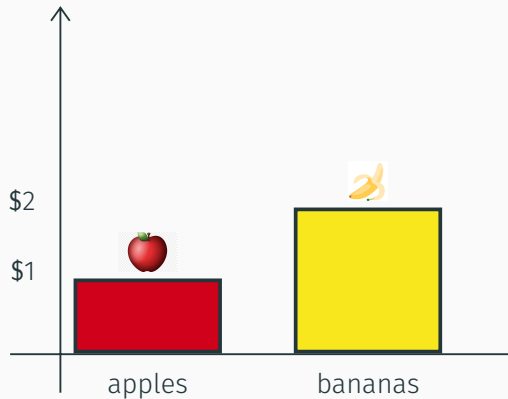
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**Prices are sticky**



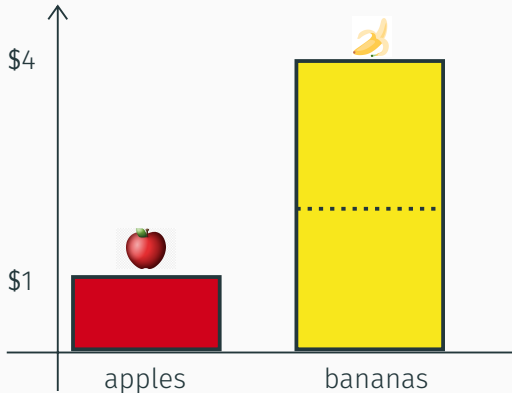


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## Prices are *sticky*

- ▶ Money supply doubles
  - ⇒ some prices are *stuck*
  - ⇒ **distorted** relative prices



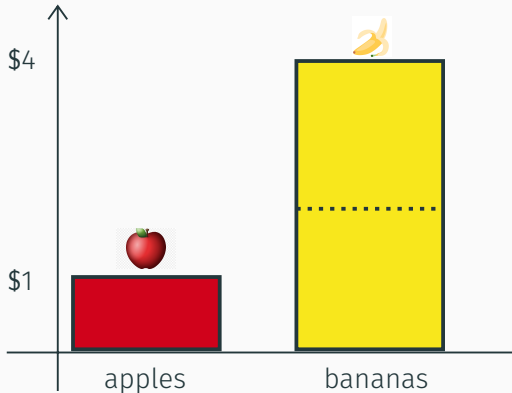
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Background: **Why does monetary policy matter?**

## Prices are *sticky*

- ▶ Money supply doubles
  - ⇒ some prices are stuck
  - ⇒ **distorted** relative prices
- ▶ Large empirical literature

▶ more



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**Textbook benchmark:** Tractable-but-unrealistic Calvo friction

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[Woodford 2003; Rubbo 2023]

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[Woodford 2003; Rubbo 2023]

## Criticism:

1. Theoretical critique: Not microfounded
2. Empirical critique: State-dependent pricing is a better fit

[Nakamura et al 2018; Cavallo and Rigobon 2016; Alvarez et al 2018; Cavallo et al 2023]

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1. **Stylized analytical model**
2. **Quantitative model**

## Related literature

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### 1. Optimal monetary policy with sectors / relative prices

- ▶ Calvo *[Rubbo 2023, Woodford 2003, Aoki 2001, Benigno 2004]*
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### 2. Menu costs, *assume* inflation targeting, solve for optimal inflation target

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### 4. Non-normative menu cost literature

- ▶ Theoretical *[Golosov-Lucas 2007; Caballero-Engel 2007; Nakamura-Steinsson 2009; Alvarez-Lippi-Paciello 2011; Midrigan 2011; Gertler-Leahy 2008; Auclert et al 2023]*
- ▶ Empirical *[Nakamura et al 2018; Cavallo-Rigobon 2016; Alvarez et al 2018; Gautier-Le Bihan 2022]*

# Roadmap

1. **Baseline model & optimal policy**
2. **Extensions**
3. **Quantitative model**
4. **Comparison to Calvo model**
5. **Conclusion and bigger picture**

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## **2. Extensions**

## **3. Quantitative model**

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## **Appendix**

# Model setup + household's problem

## General setup:

- ▶ Off-the shelf sectoral model with  $S$  sectors
- ▶ Each sector is a continuum of firms, bundled with CES technology
- ▶ Static model (& no linear approximation)

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## Household's problem:

$$\max_{C, N, M} \ln(C) - N + \ln\left(\frac{M}{P}\right)$$

$$\text{s.t. } PC + M = WN + D + M_{-1} - T$$

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## Optimality conditions:

$$\begin{aligned} c_i &= \frac{1}{S} \frac{PC}{p_i} \\ PC &= M \\ W &= M \end{aligned}$$

**Technology:** In given sector  $i$ , continuum of firms  $j \in [0, 1]$  with technology

$$y_i(j) = A_i \cdot n_i(j)$$

**Demand:**  $y_i(j) = y_i \left( \frac{p_i(j)}{p_i} \right)^{-\eta}$



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$$\left( p_i y_i - \frac{W}{A_i} y_i (1 - \tau) \right) - W \psi \chi_i$$

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- ▶  $\chi_i$ : indicator for price change vs. not

# Intermediate firms: price setting with menu costs

► more production structure

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⇒ **Direct cost of menu costs:** excess disutility of labor

$$N = \sum_i n_i + \psi \sum_i \chi_i$$

- Other specifications do not affect result

► more

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# Menu costs induce an inaction region

Objective function of sector  $i$  firm:  $\left( p_i y_i - \frac{W}{A_i} y_i (1 - \tau) \right) - W \psi \chi_i$



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**Inaction region:** don't adjust iff  $p_i^* = \frac{W}{A_i}$  close to  $p_i^{\text{old}}$

# Optimal policy after a productivity shock

► Formal planner's problem

- Start at steady state: all sectors have  $A_i^{ss} = 1 \quad \forall i$ , so  $p_i^{ss} = W^{ss} \equiv 1$

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**Proposition 1:** there exists a threshold level of productivity  $\bar{A}$  s.t.:

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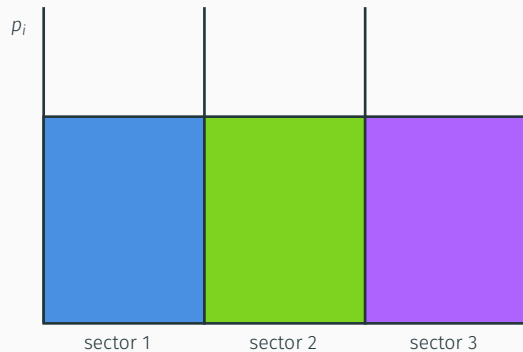
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$$p_i = p_i^{ss} \quad \forall i$$

Recall:  $p_i^* = MC_i = \frac{W}{A_i}$



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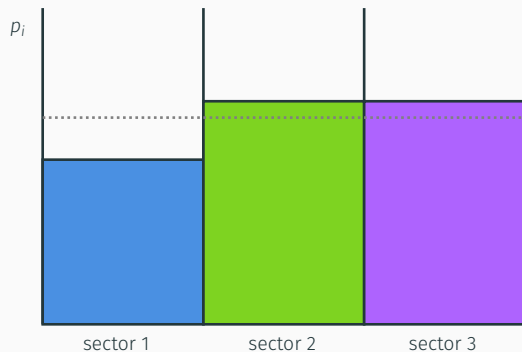


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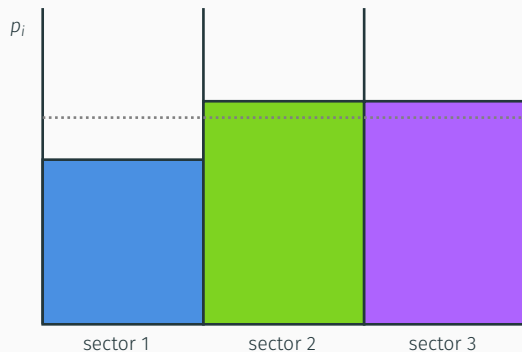
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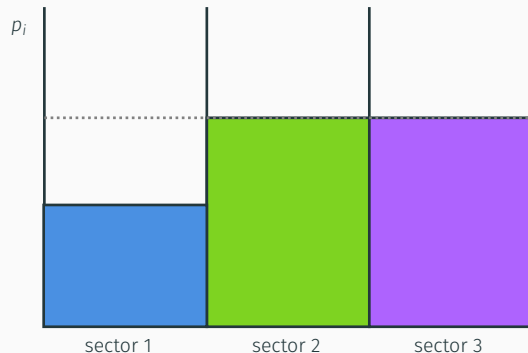


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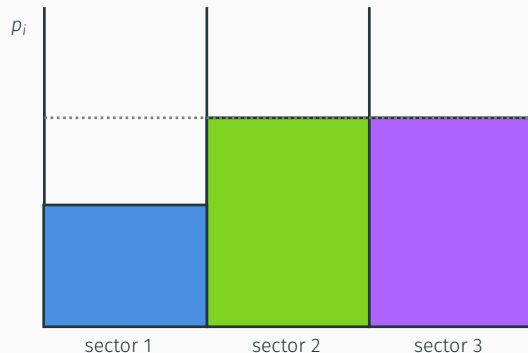
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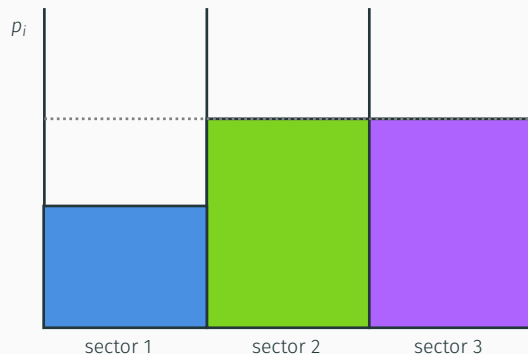
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[math](#)[more math](#)

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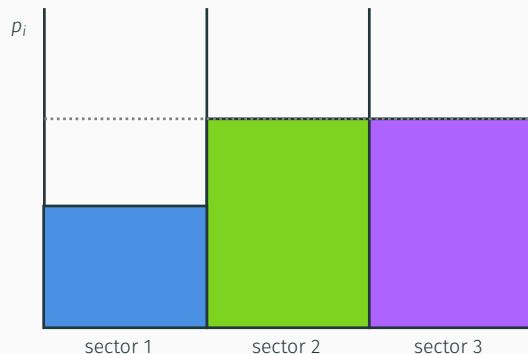
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**Stabilize nominal MC of  
unshocked firms**

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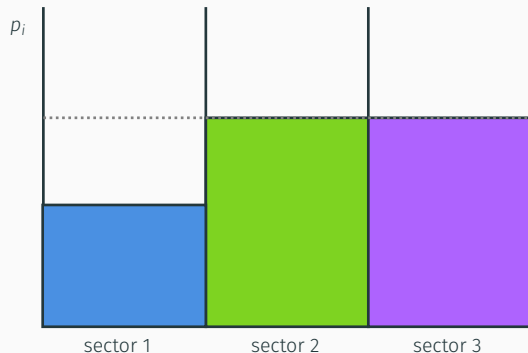
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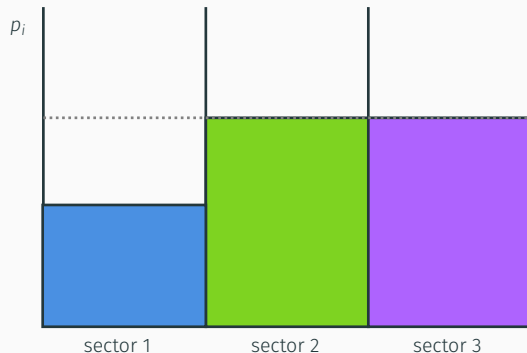
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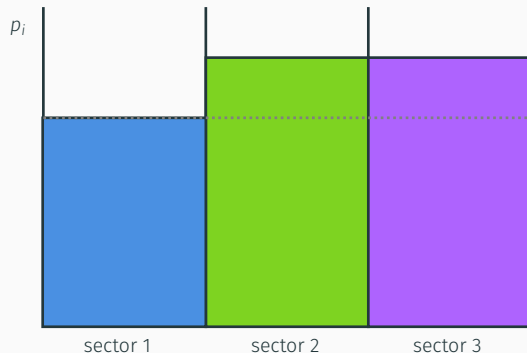


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**Only sectors  $k$  adjusts**

$$W^f - (S - 1)\psi$$

## Small shocks: state dependence of optimal policy

[▸ math](#)[▸ more math](#)

	Sectors $k$ adjust	Sectors $k$ not adjust
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**Lemma 2:**  $\exists \bar{A}$  such that

$$\mathbb{W}_{\text{only 1 adjusts}} > \mathbb{W}_{\text{none adjust}}$$

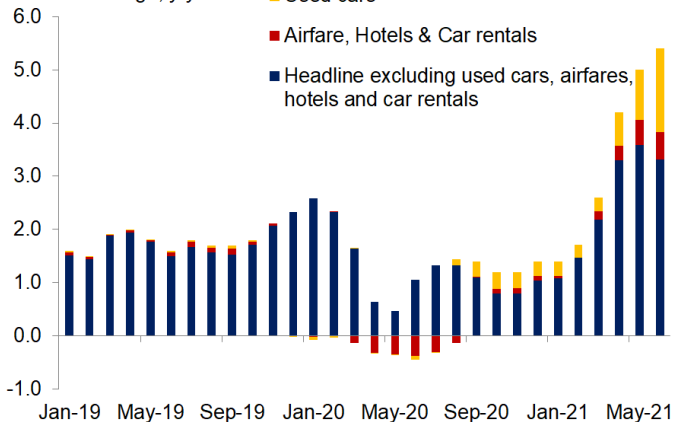
iff  $A_1 > \bar{A}$ . Furthermore,  $\bar{A}$  is increasing in  $\psi$ .

# Interpretation: “looking through” shocks

## Example 1: used cars (2021)

### US: Consumer price index (CPI)

Percent change, y/y



Source : Oxford Economics/BLS

# Interpretation: “looking through” shocks



**Example 2:** energy shock (2022)

## Looking through higher energy prices? Monetary policy and the green transition

Isabel Schnabel, Member of the ECB's Executive Board



# The welfare loss of inflation targeting

**“Inflation targeting”:**  $P = P^{SS}$  (while having correct relative prices)

**Proposition 2:** Suppose  $A_1 > \bar{A}$ . Then:

1. Inflation targeting requires all sectors adjust their prices
2. Welfare loss from inflation targeting  $\propto$  size of menu costs

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What are menu costs?

1. **Physical adjustment costs.** Baseline interpretation.
2. **Information costs.** Fixed costs of information acquisition / processing.
  - Results unchanged
3. **Behavioral costs.** Consumer *distaste* for price changes.
  - Results unchanged

## How large are menu costs?

**Summary:** at least 0.5% of firm revenues, plausibly much more

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**2. Direct measurement.** For *physical* adjustment costs,

Levy et al (1997, QJE): 5 grocery chains

- ▶ 0.7% revenue

Dutta et al (1999, JMCB): drugstore chain

- ▶ 0.6% revenue

Zbaracki et al (2003, Restat): mfg

- ▶ 1.2% revenue

**1. Baseline model & optimal policy**

**2. Extensions**

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**Appendix**

## Generalized model: stabilize nominal MC of unshocked firms

Generalized model:

1. Any (HOD1) aggregator:

$$C = F(c_1, \dots, c_S)$$

2. Potentially DRS production

technology:  $y_i(j) = A_i n_i(j)^{1/\alpha}$  with  
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**Nominal MC:**

$$MC_i(j) = \left[ \alpha \frac{W}{A_i^\alpha} (y_i p_i^\eta)^{\alpha-1} \right]^\theta$$
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Proposition 1 extended: optimal policy stabilizes **nominal marginal costs of unshocked firms**

$$\implies Y \uparrow, P \downarrow$$

## “Macro functional forms”

More general example:

1.  $C = \prod c_i^{1/S}$

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Proposition 1 extended: optimal policy stabilizes **nominal marginal costs of unshocked firms**

$\implies$  stabilize a weighted average of wages and prices,  $W^\lambda P^{1-\lambda}$

# Production networks: stabilize a weighted average of $P$ and $W$

## Baseline model:

- Production technology:

$$y_i = A_i n_i$$

## Roundabout production network:

- Production technology:

$$y_i = A_i n_i^\beta l_i^{1-\beta}$$
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**Proposition 3:** Consider any shock not affecting relative prices, e.g. a perfectly uniform shock:  $A_1 = \dots = A_S \equiv A$ .

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- ▶ Relative prices don't need to change

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*Proof idea:*

- Relative prices don't need to change
- Stable prices thus guarantee:
  1. Correct relative prices
  2. Zero direct costs

## Additional extensions

1. Under sticky wages due to menu costs, optimal policy still stabilizes  $W$ ;

▶ more

2. Optimal policy is not about selection effects: a CalvoPlus model

▶ more

3. Heterogeneity across sectors: a monetary “least-cost avoider” principal

▶ more

1. Baseline model & optimal policy

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Appendix

# Quantitative model: setup and solution method

**Dynamic** model with **idiosyncratic** + sectoral shocks

## Household

$$\begin{aligned} \max_{\{C_t, N_t, B_t, M_t\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\gamma}}{1-\gamma} - \omega \frac{N_t^{1+\varphi}}{1+\varphi} + \ln \left( \frac{M_t}{P_t} \right) \right] \\ \text{s.t.} \quad & P_t C_t + B_t + M_t \leq R_t B_{t-1} + W_t N_t + M_{t-1} + D_t - T_t \end{aligned}$$

## Intermediate firms

$$\begin{aligned} \max_{p_{it}(j), \chi_{it}(j)} \quad & \sum_{t=0}^{\infty} \mathbb{E} \left[ \frac{1}{R^t P_t} \{ p_{it}(j) y_{it}(j) - W_t n_{it}(j) (1 - \tau) - \chi_{it}(j) \psi W_t \} \right] \\ \text{s.t.} \quad & y_{it}(j) = A_{it} a_{it}(j) n_{it}(j)^\alpha \quad \text{and} \quad R^t = \prod_{\tau=0}^t R_\tau \end{aligned}$$

where idiosyncratic productivity follows an AR(1)

$$\log(a_{it}(j)) = \rho_{\text{idio}} \log(a_{it-1}(j)) + \varepsilon_t^{\text{idio}}$$

# Calibration

Two sets of parameters to calibrate:

(1) standard or drawn from literature and

	Parameter (quarterly frequency)	Value	Target
$\beta$	Discount factor	0.99	standard
$\omega$	Disutility of labor	1	standard
$\varphi$	Inverse Frisch elasticity	0	Golosov-Lucas 2007
$\gamma$	Inverse EIS	2	standard
$S$	Number of sectors	6	Nakamura-Steinsson 2010
$\eta$	Elasticity of subst. between sectors	5	standard value
$\alpha$	Returns to scale	0.6	standard value



# Calibration

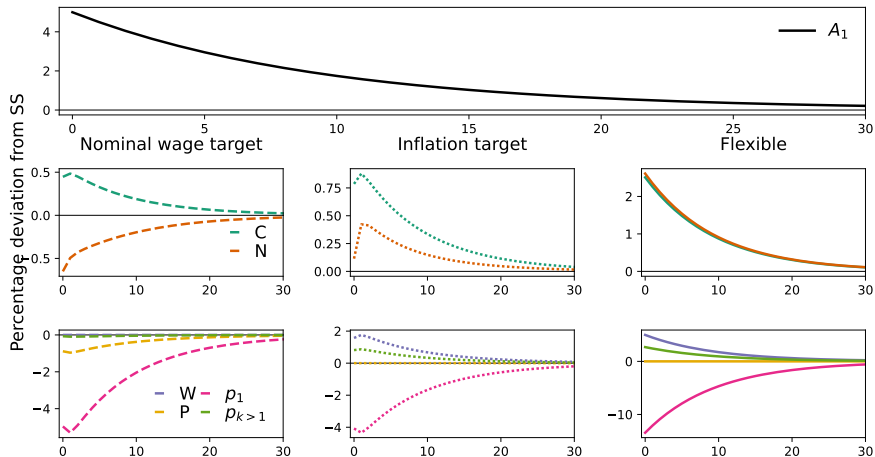
Two sets of parameters to calibrate:

(1) standard or drawn from literature and (2) calibrated by SMM targeting

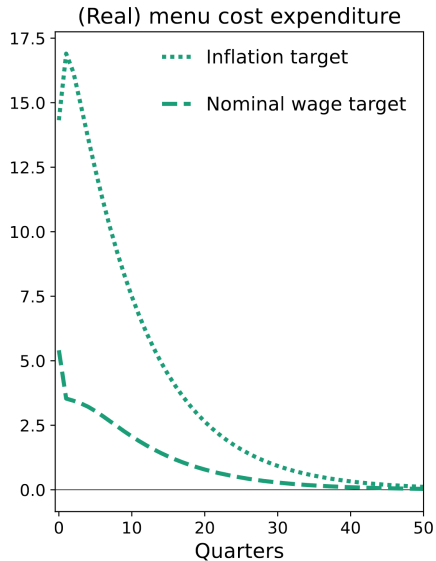
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$\alpha$	Returns to scale	0.6	standard value
$\sigma_{\text{idio}}$	Std. of idio. shocks	0.13	menu cost expenditure / revenue $\sim 1\%$ and share of price changers $\sim 26.1\%$
$\rho_{\text{idio}}$	Persistence of idio. shocks	0.86	
$\psi$	Menu cost	0.016	

# Exercise: perfect foresight sectoral shock

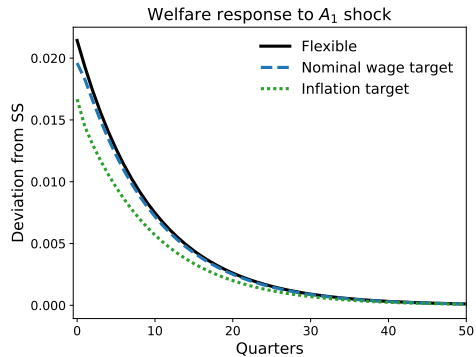
Impulse responses after  $A_1$  shock



## Policy comparison: menu cost expenditure

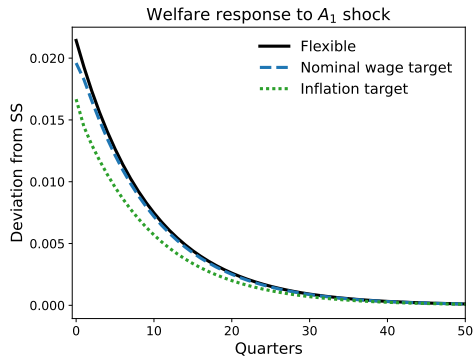


# Policy comparison: welfare

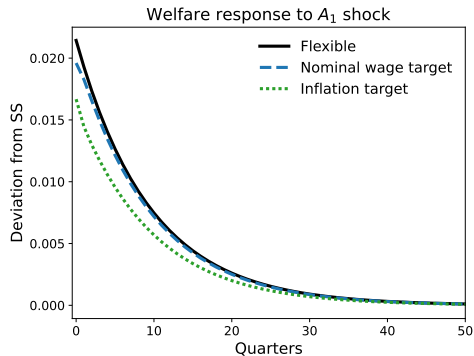


# Policy comparison: welfare

## 1. Consider **welfare** under $W$ targeting



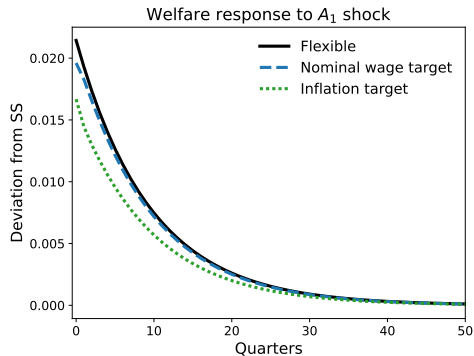
# Policy comparison: welfare



1. Consider welfare under  $W$  targeting
2. How much extra  $C$  is needed to match welfare under flexible prices?

$$\sum_t \beta^t U((1 + \lambda)C_t, N_t)$$
$$= \sum_t \beta^t U(C_t^{\text{flex}}, N_t^{\text{flex}})$$

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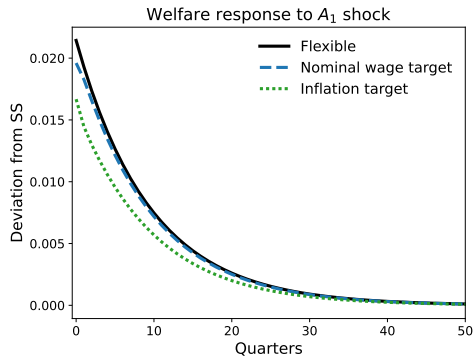


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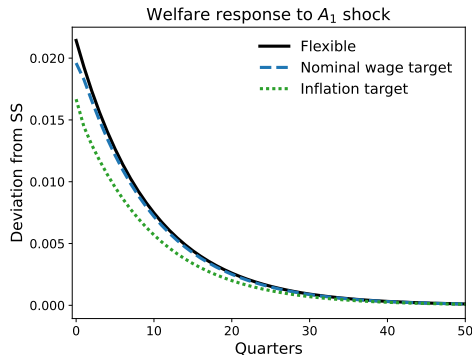
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$$\lambda^W = 0.004\%$$

$$\lambda^P = 0.02\%$$



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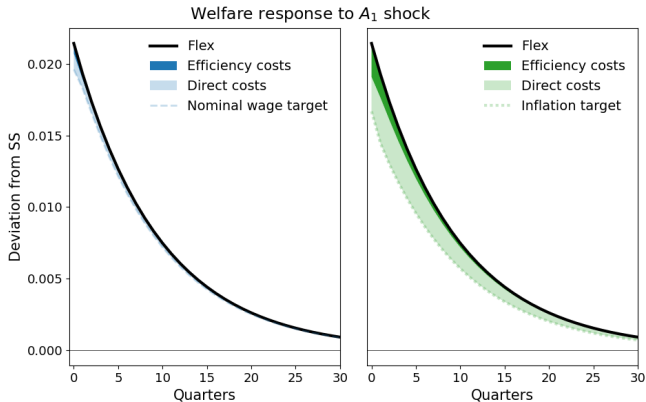
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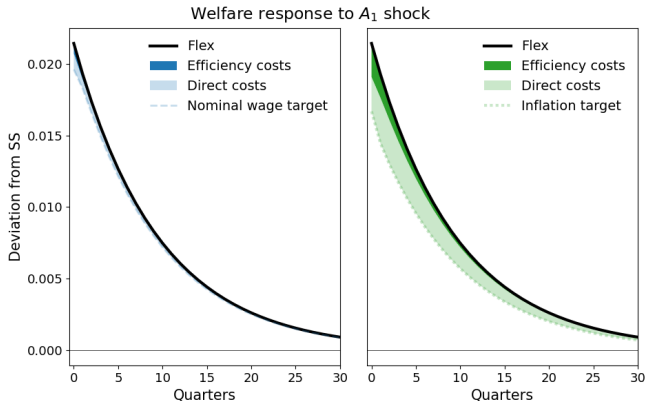
$\Rightarrow$  **welfare loss of sticky prices -80.6%**

# Decomposing welfare



1. **Direct costs:**  $\psi\chi_t$ , disutility of labor from menu costs
2. **Efficiency costs:** welfare loss from incorrect relative prices

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1. **Direct costs:**  $\psi\chi_t$ , disutility of labor from menu costs

2. **Efficiency costs:** welfare loss from incorrect relative prices

- ▶ Direct costs:  $\tilde{\lambda}^W = 0.0007\%$  and  $\tilde{\lambda}^P = 0.0060\%$
- ▶ Recall total welfare losses:  $\lambda^W = 0.0040\%$  and  $\lambda^P = 0.0200\%$
- ▶ **Interpretation:** welfare improvement comes from both channels

## Numerically-optimal policy in simple class of rules

Consider monetary policy  
rules stabilizing:

$$W^\xi P^{1-\xi}$$

$$\xi \in [0, 1]$$

Recall  $\lambda$ : “how much extra  $C$   
needed to match welfare  
response of flex-price  
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# Numerically-optimal policy in simple class of rules

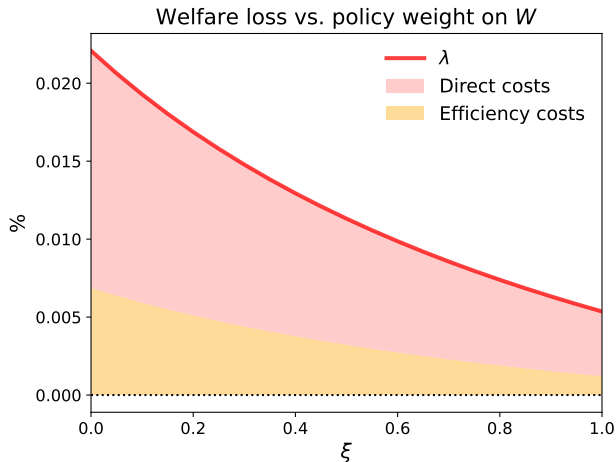
Consider monetary policy rules stabilizing:

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$$\xi \in [0, 1]$$

Recall  $\lambda$ : “how much extra  $C$  needed to match welfare response of flex-price economy?”

## Numerically-optimal policy: Stabilize $W$ alone



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- ▶ **Multisector Calvo optimal policy: inflation targeting,  $P = P^{ss}$ .** Why?

*[Woodford; Rubbo; Aoki; cf Guerrieri-Lorenzoni-Straub-Werning]*

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$$N = \sum n_i + \psi \sum \mathbb{I}\{p_i \neq p_i^{ss}\}$$

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# Why not inflation targeting?

[▶ more](#)

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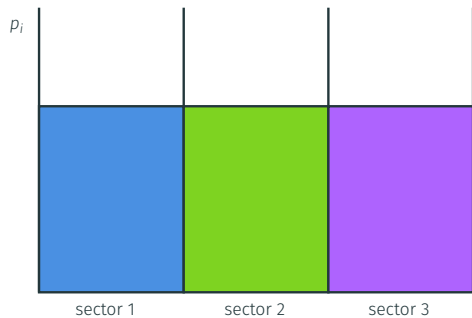
**Convex costs  $\implies$  smooth price changes across sectors**

**Calvo:** Likewise, *welfare cost of price dispersion is convex*:

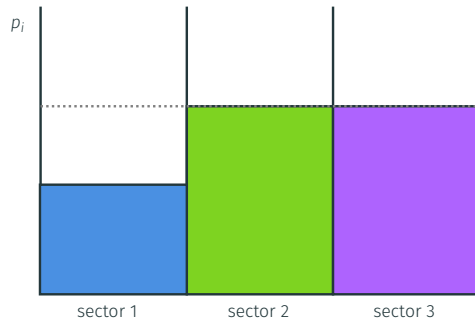
$$\Delta \equiv \sum_{i=1}^S \int_0^1 \left[ \frac{p_i(j)}{p_i} \right]^{-\eta} dj$$

where  $\eta > 1$  is the within-sector elasticity of substitution

# Calvo diagram: shocking sector-1 productivity

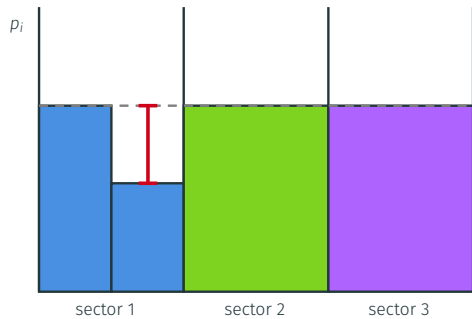


**Steady state**



**Flexible prices, after shock**

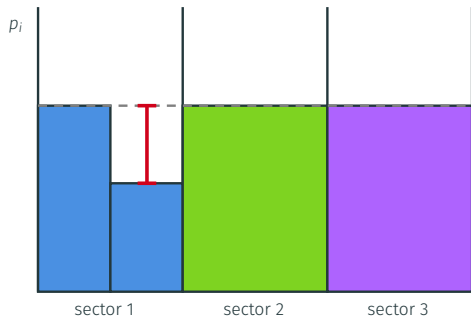
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**Nominal wage targeting  
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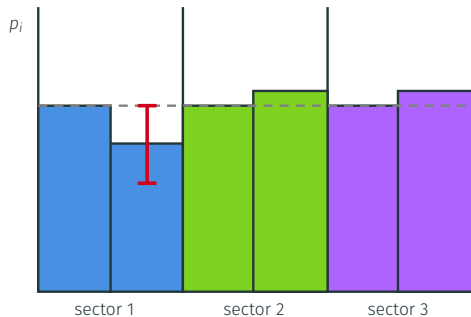
*Lots of price dispersion: only one sector*

# Calvo diagram: shocking sector-1 productivity

[▶ math](#)

**Nominal wage targeting  
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*Lots of price dispersion: only one sector*



**Inflation targeting  
under Calvo**

*Little price dispersion: all sectors*



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**Appendix**

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- ▶ RBC + Calvo = inflation targeting
- ▶ RBC + menu costs = countercyclical inflation

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4. Sticky prices [**new**]: **Caratelli and Halperin (2024)**

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In baseline menu cost model, **inflation should be countercyclical** after sectoral shocks

Rationale:

- ▶ Inflation targeting **forces firms to adjust unnecessarily**, which is costly with menu costs
- ▶ Nominal wage targeting does not

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Rationale:

- ▶ Inflation targeting **forces firms to adjust unnecessarily**, which is costly with menu costs
- ▶ Nominal wage targeting does not

**Future work:**

- ▶ Convexity of menu costs
- ▶ Better direct measurement of menu costs
- ▶ “Unified theory of optimal monetary policy”?

Thank you!

**1. Baseline model & optimal policy**

**2. Extensions**

**3. Quantitative model**

**4. Comparison to Calvo model**

**5. Conclusion and bigger picture**

**Appendix**

## Sectoral packagers:

$$y_i = \left[ \int_0^1 y_i(j)^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}}$$
$$y_i(j) = y_i \left[ \frac{p_i(j)}{p_i} \right]^{-\eta}$$
$$p_i = \left[ \int_0^1 p_i(j)^{1-\eta} dj \right]^{\frac{1}{1-\eta}}$$

## Intermediate producers:

$$y_i(j) = A_i n_i(j)$$
$$p_i(j)^{\text{opt}} = \frac{\eta}{\eta-1} (1-\tau) \frac{W}{A_i}$$
$$\chi_i = \mathbb{I} \left\{ \frac{1}{\eta} > y_i \left[ \frac{p_i^{\text{old}}}{p_i} \right]^{-\eta} \left( p_i^{\text{old}} - \frac{W}{A_i} \frac{\eta-1}{\eta} \right) \right\}$$

## Household:

$$M = PC$$

$$M = W$$

$$C = \prod C_i^{1/S}$$

$$P = S \prod p_i^{1/S}$$

## Government:

$$1 - \tau = \frac{\eta - 1}{\eta}$$

$$-T + (M - M_{-1}) = \tau W \sum n_i$$

## Market clearing:

$$N = \sum n_i + \psi \sum \chi_i$$

**Final goods demand:**

$$C = \prod y_i^{1/S}$$

$$P = S \prod p_i^{1/S}$$

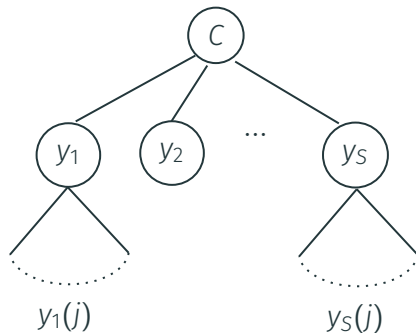
$$y_i = \frac{1}{S} \frac{PC}{p_i}$$

**Sectoral packagers** (competitive):

$$y_i = \left[ \int_0^1 y_i(j)^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}}$$

$$y_i(j) = y_i \left[ \frac{p_i(j)}{p_i} \right]^{-\eta}$$

$$p_i = \left[ \int_0^1 p_i(j)^{1-\eta} dj \right]^{\frac{1}{1-\eta}}$$



# Equilibrium in four possible regimes

[▶ back](#)

	Sectors $k$ adjust	Sectors $k$ not adjust
Sector 1 adjusts	.	.
Sector 1 not adjust	.	.



# Equilibrium in four possible regimes

[▶ back](#)

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Flexible price benchmark ( $\psi = 0$ ):

# Equilibrium in four possible regimes

[▶ back](#)

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## Flexible price benchmark ( $\psi = 0$ ):

- ▶  $p_1 = \frac{W}{A_1}$  and  $p_k = W$
- ▶ Key object: relative price

$$\left(\frac{p_1}{p_k}\right)_{\text{flex}} = \frac{1}{A_1}$$

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- ▶ Flex-price welfare:

$$\mathbb{W}_{\text{flex}} = \ln(C_{\text{flex}}) - N_{\text{flex}}$$

# Equilibrium in four possible regimes

[▶ back](#)

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## All adjust:

# Equilibrium in four possible regimes

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# Equilibrium in four possible regimes

[▶ back](#)

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- ▶  $C = C_{\text{flex}}$ ; and  $N = N_{\text{flex}} + S\psi$

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- ▶ Welfare:  $\mathbb{W}_{\text{all adjust}} = \mathbb{W}_{\text{flex}} - S\psi$



# Equilibrium in four possible regimes

	Sectors $k$ adjust	Sectors $k$ not adjust
Sector 1 adjusts	$\mathbb{W}_{\text{flex}} - S\psi$	.
Sector 1 not adjust	.	.

## Flexible price benchmark ( $\psi = 0$ ):

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$$\left(\frac{p_1}{p_k}\right)_{\text{all adjust}} = \frac{1}{A_1} = \left(\frac{p_1}{p_k}\right)_{\text{flex}}$$

- ▶  $C = C_{\text{flex}}$ ; and  $N = N_{\text{flex}} + S\psi$
- ▶ Welfare:  $\mathbb{W}_{\text{all adjust}} = \mathbb{W}_{\text{flex}} - S\psi$

## Equilibrium in four possible regimes (2)

	Sectors $k$ adjust	Sectors $k$ not adjust
Sector 1 adjusts	$\mathbb{W}_{\text{flex}} - S\psi$	.
Sector 1 not adjust	.	.

### Flexible price benchmark ( $\psi = 0$ ):

- ▶  $p_1 = \frac{W}{A_1}$  and  $p_k = W$
- ▶ Key object: relative price

$$\left(\frac{p_1}{p_k}\right)_{\text{flex}} = \frac{1}{A_1}$$

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## Equilibrium in four possible regimes (2)

	Sectors $k$ adjust	Sectors $k$ not adjust
Sector 1 adjusts	$\mathbb{W}_{\text{flex}} - S\psi$	.
Sector 1 not adjust	.	.

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 $\mathbb{W}_{\text{flex}} = \ln(C_{\text{flex}}) - N_{\text{flex}}$

### Only sector 1 adjusts:

- ▶  $p_1 = \frac{W}{A_1}$  and  $p_k = p_k^{\text{ss}} = 1$

## Equilibrium in four possible regimes (2)

	Sectors $k$ adjust	Sectors $k$ not adjust
Sector 1 adjusts	$\mathbb{W}_{\text{flex}} - S\psi$	.
Sector 1 not adjust	.	.

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$$\left(\frac{p_1}{p_k}\right) = \frac{W}{A_1}$$

## Equilibrium in four possible regimes (2)

	Sectors $k$ adjust	Sectors $k$ not adjust
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 $\mathbb{W}_{\text{flex}} = \ln(C_{\text{flex}}) - N_{\text{flex}}$

### Only sector 1 adjusts:

- ▶  $p_1 = \frac{W}{A_1}$  and  $p_k = p_k^{\text{ss}} = 1$
- ▶ Relative price:

$$\left(\frac{p_1}{p_k}\right) = \frac{W}{A_1}$$

- ▶ Replicate flex-price relative price by:  
 setting  $W = W^{\text{ss}} = 1$

## Equilibrium in four possible regimes (2)

	Sectors $k$ adjust	Sectors $k$ not adjust
Sector 1 adjusts	$\mathbb{W}_{\text{flex}} - S\psi$	$\mathbb{W}_{\text{flex}} - \psi$
Sector 1 not adjust	.	.

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- ▶  $p_1 = \frac{W}{A_1}$  and  $p_k = W$
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- ▶ Relative price:

$$\left(\frac{p_1}{p_k}\right) = \frac{W}{A_1}$$

- ▶ Replicate flex-price relative price by:  
 setting  $W = W^{\text{ss}} = 1$
- ▶ Welfare under optimal policy:  
 $\mathbb{W}_{\text{only 1 adjusts}} = \mathbb{W}_{\text{flex}} - \psi$

## Equilibrium in four possible regimes (2)

	Sectors $k$ adjust	Sectors $k$ not adjust
Sector 1 adjusts	$\mathbb{W}_{\text{flex}} - S\psi$	$\mathbb{W}_{\text{flex}} - \psi$
Sector 1 not adjust	$\mathbb{W}_{\text{flex}} - (S-1)\psi$	.

### Flexible price benchmark ( $\psi = 0$ ):

- ▶  $p_1 = \frac{W}{A_1}$  and  $p_k = W$
- ▶ Key object: relative price

$$\left(\frac{p_1}{p_k}\right)_{\text{flex}} = \frac{1}{A_1}$$

- ▶  $C_{\text{flex}} = A_1^{1/S}/S$ ; and  $N_{\text{flex}} = 1$
- ▶ Flex-price welfare:  
 $\mathbb{W}_{\text{flex}} = \ln(C_{\text{flex}}) - N_{\text{flex}}$

### Only sectors $k$ adjust:

- ▶ Symmetric.

## Equilibrium in four possible regimes (3)

	Sectors $k$ adjust	Sectors $k$ not adjust
Sector 1 adjusts	$\mathbb{W}_{\text{flex}} - S\psi$	$\mathbb{W}_{\text{flex}} - \psi$
Sector 1 not adjust	$\mathbb{W}_{\text{flex}} - (S - 1)\psi$	.

### Flexible price benchmark ( $\psi = 0$ ):

- ▶  $p_1 = \frac{W}{A_1}$  and  $p_k = W$
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$$\left(\frac{p_1}{p_k}\right)_{\text{flex}} = \frac{1}{A_1}$$

- ▶  $C_{\text{flex}} = A_1^{1/S}/S$ ; and  $N_{\text{flex}} = 1$
- ▶ Flex-price welfare:  
 $\mathbb{W}_{\text{flex}} = \ln(C_{\text{flex}}) - N_{\text{flex}}$

### None adjust:

- ▶  $p_1 = p_1^{ss} = 1$  and  $p_k = p_k^{ss} = 1$



## Equilibrium in four possible regimes (3)

	Sectors $k$ adjust	Sectors $k$ not adjust
Sector 1 adjusts	$\mathbb{W}_{\text{flex}} - S\psi$	$\mathbb{W}_{\text{flex}} - \psi$
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### Flexible price benchmark ( $\psi = 0$ ):

- ▶  $p_1 = \frac{W}{A_1}$  and  $p_k = W$
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- ▶ Flex-price welfare:  
 $\mathbb{W}_{\text{flex}} = \ln(C_{\text{flex}}) - N_{\text{flex}}$

### None adjust:

- ▶  $p_1 = p_1^{\text{ss}} = 1$  and  $p_k = p_k^{\text{ss}} = 1$
- ▶ Relative price:

$$\left(\frac{p_1}{p_k}\right) = 1 \neq \left(\frac{p_1}{p_k}\right)_{\text{flex}}$$

## Equilibrium in four possible regimes (3)

	Sectors $k$ adjust	Sectors $k$ not adjust
Sector 1 adjusts	$\mathbb{W}_{\text{flex}} - S\psi$	$\mathbb{W}_{\text{flex}} - \psi$
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- ▶ Relative price:

$$\left(\frac{p_1}{p_k}\right) = 1 \neq \left(\frac{p_1}{p_k}\right)_{\text{flex}}$$

- ▶ Cannot replicate flex-price

## Equilibrium in four possible regimes (3)

	Sectors $k$ adjust	Sectors $k$ not adjust
Sector 1 adjusts	$\mathbb{W}_{\text{flex}} - S\psi$	$\mathbb{W}_{\text{flex}} - \psi$
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- ▶ Cannot replicate flex-price
- ▶ **Upside:** no menu costs!

## Equilibrium in four possible regimes (3)

	Sectors $k$ adjust	Sectors $k$ not adjust
Sector 1 adjusts	$\mathbb{W}_{\text{flex}} - S\psi$	$\mathbb{W}_{\text{flex}} - \psi$
Sector 1 not adjust	$\mathbb{W}_{\text{flex}} - (S - 1)\psi$	$-\ln(S - 1 + 1/A_1) - 1$

### Flexible price benchmark ( $\psi = 0$ ):

- ▶  $p_1 = \frac{W}{A_1}$  and  $p_k = W$
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 $\mathbb{W}_{\text{flex}} = \ln(C_{\text{flex}}) - N_{\text{flex}}$

### None adjust:

- ▶  $p_1 = p_1^{\text{ss}} = 1$  and  $p_k = p_k^{\text{ss}} = 1$
- ▶ Relative price:

$$\left(\frac{p_1}{p_k}\right) = 1 \neq \left(\frac{p_1}{p_k}\right)_{\text{flex}}$$

- ▶ Cannot replicate flex-price
- ▶ Upside: no menu costs!
- ▶ Welfare:

$$\mathbb{W}_{\text{none adjust}} = -\ln(S - 1 + 1/A_1) - 1$$

# Proving optimal policy

	Sectors $k$ adjust	Sectors $k$ not adjust
Sector 1 adjusts	$\mathbb{W}_{\text{flex}} - S\psi$	$\mathbb{W}_{\text{flex}} - \psi$
Sector 1 not adjust	$\mathbb{W}_{\text{flex}} - (S - 1)\psi$	$-\ln(S - 1 + 1/A_1) - 1$

**Lemma 1:** If adjusting, only shocked sectors should adjust

$$\mathbb{W}_{\text{only 1 adjusts}} > \mathbb{W}_{\text{all adjust}}, \mathbb{W}_{\text{only } k \text{ adjust}}$$

	Sectors $k$ adjust	Sectors $k$ not adjust
Sector 1 adjusts	$\mathbb{W}_{\text{flex}} - S\psi$	$\mathbb{W}_{\text{flex}} - \psi$
Sector 1 not adjust	$\mathbb{W}_{\text{flex}} - (S - 1)\psi$	$-\ln(S - 1 + 1/A_1) - 1$

**Lemma 1:** If adjusting, only shocked sectors should adjust

$$\mathbb{W}_{\text{only 1 adjusts}} > \mathbb{W}_{\text{all adjust}}, \mathbb{W}_{\text{only } k \text{ adjust}}$$

**Lemma 2:**  $\exists \bar{A}$  such that

$$\mathbb{W}_{\text{only 1 adjusts}} > \mathbb{W}_{\text{none adjust}}$$

iff  $A_1 > \bar{A}$ . Furthermore,  $\bar{A}$  is increasing in  $\psi$ .

## Formally: Social planner's problem

[▶ back](#)

$$\max_{X \in \{A, B, C, D\}} \mathbb{U}^X$$

$$\mathbb{U}^A = \left\{ \begin{array}{l} \max_M \quad \ln[M] - M[S - 1 + 1/\gamma] \\ \text{s.t.} \quad \min(\gamma\lambda_1, \lambda_2) \leq M \leq \max(\gamma\lambda_1, \lambda_2) \end{array} \right\}$$

$$\mathbb{U}^B = \left\{ \ln \left[ \frac{1}{S} \gamma^{1/S} \right] - 1 - \psi \right\}$$

$$\mathbb{U}^C = \left\{ \begin{array}{l} \max_M \quad \ln \left[ \left( \frac{\gamma}{S} \right)^{\frac{1}{S}} \cdot M^{\frac{S-1}{S}} \right] - \left[ (S-1)M + \frac{1}{S} \right] - \frac{1}{S} \psi \\ \text{s.t.} \quad \lambda_1 < M < \min(\gamma\lambda_1, \lambda_2) \end{array} \right\}$$

$$\mathbb{U}^D = \left\{ \begin{array}{l} \max_M \quad \ln \left[ S^{\frac{1-S}{S}} M^{\frac{1}{S}} \right] - \left[ \frac{S-1}{S} + \frac{M}{\gamma} \right] - \frac{S-1}{S} \psi \\ \text{s.t.} \quad \max(\gamma\lambda_1, \lambda_2) < M < \gamma\lambda_2 \end{array} \right\}$$

$$\text{where } \lambda_1 = \frac{1}{S} (1 - \sqrt{\psi}), \quad \lambda_2 = \frac{1}{S} (1 + \sqrt{\psi})$$

Example: Social planner's *constrained* problem for “neither adjust”

$$\max_M U(C(M), N(M)) \quad (1)$$

$$\text{s.t. } D_1^{\text{adjust}} < D_1^{\text{no adjust}} \quad (2)$$

$$D_k^{\text{adjust}} < D_k^{\text{no adjust}} \quad (3)$$

$$\implies M_{\text{unconstrained}}^*$$

Social planner's *unconstrained* problem: maximize (1), without constraints

$$\implies M_{\text{constrained}}^*$$

**Adjustment externality:**  $M_{\text{unconstrained}}^* \neq M_{\text{constrained}}^*$



**Labor costs:** Welfare mechanism is *higher labor*

$$\begin{aligned} & \text{profits}_i - W\psi \cdot \chi_i \\ \implies N &= \sum n_i + \psi \sum \chi_i \end{aligned}$$

**Real resource cost:** Welfare mechanism is *lower consumption*

$$\begin{aligned} & \text{profits}_i \cdot (1 - \psi \cdot \chi_i) \\ \implies C &= Y \left( 1 - \psi \sum_i \chi_i \right) \end{aligned}$$

**Direct utility cost:** Welfare mechanism is *direct*

$$\text{utility} - \psi \cdot \sum \chi_i$$

Recall:

$$p_i = \frac{W}{A_i}$$

Suppose  $A_i \uparrow$ . Then either:

1.  $p_i \downarrow$
2.  $W \uparrow$ 
  - But then  $p_j \uparrow$

Suppose  $A_i \downarrow$ . Then either:

1.  $p_i \uparrow$
2.  $W \downarrow$ 
  - But at least then  $p_j \downarrow$

Nominal wage targeting:

$$\hat{W} = 0$$

$$\hat{p}_1(A) = -\hat{\gamma}$$

$$\hat{p}_k(A) = 0$$

$$\hat{P} = -\frac{1}{S}(1 - \theta)\hat{\gamma}$$

$$\hat{C} = \frac{1}{S}(1 - \theta)\hat{\gamma}$$

$$\hat{N} = -\frac{1}{S}\theta\hat{\gamma}$$

Inflation targeting:

$$\hat{W} = \frac{\hat{\gamma}}{S}$$

$$\hat{p}_1(A) = -\hat{\gamma} + \frac{1}{S}\hat{\gamma}$$

$$\hat{p}_k(A) = \frac{\hat{\gamma}}{S}$$

$$\hat{P} = 0$$

$$\hat{C} = \hat{C}^f = \frac{\hat{\gamma}}{S}$$

$$\hat{N} = \hat{N}^f = 0$$

# “Generalized multisector Rotemberg”

► back

## Calvo is isomorphic to Rotemberg menu cost model (Nisticò 2007)

- ▶ Rotemberg quadratic menu costs:

$$\psi \cdot (p_i - p_i^{ss})^2 \mathbb{I}\{p_i \neq p_i^{ss}\}$$

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- ▶ Contrast with *nonconvex* menu costs:

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**Difference in optimal policy comes from convexity**

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## Difference in optimal policy comes from **convexity**

- ▶ Rotemberg labor market clearing:

$$N = \sum n_i + \psi \sum (p_i - p_i^{ss})^2 \mathbb{I}\{p_i \neq p_i^{ss}\}$$

- ▶ Nonconvex labor market clearing:

$$N = \sum n_i + \psi \sum \mathbb{I}\{p_i \neq p_i^{ss}\}$$



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$$\rightarrow \sum n_i + \psi \sum \mathbb{I}\{p_i \neq p_i^{ss}\}$$

- ▶ Nonconvex labor market clearing:

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## Sticky prices model:

differentiated output + homogenous labor

$$p_1 = \frac{W}{A_1}$$

$$p_k = \frac{W}{A_k}$$

With shock to  $A_1$ , want:

- ▶  $p_1$  adjusts
- ▶  $W$  stabilized, so  $p_k$  doesn't have to change

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## **Monopsony** sticky wage model:

homogeneous output + differentiated labor

$$P = \frac{W_1}{A_1}$$

$$P = \frac{W_k}{A_k}$$

$$W_1 = W_k$$

With shock to  $A_1$ , want:

- ▶  $P$  adjust, so  $W_1 = W_k$  doesn't have to adjust

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$$W_1 = W_k$$

With shock to  $A_1$ , want:

- ▶  $P$  adjust, so  $W_1 = W_k$  doesn't have to adjust

**Monopsony model is anti-Keynesian:** inverted NKPC (Rowe 2014; Dennerly 2021)

## Sticky prices model:

differentiated output + homogenous labor

$$p_1 = \frac{W}{A_1}$$

$$p_k = \frac{W}{A_k}$$

With shock to  $A_1$ , want:

- ▶  $p_1$  adjusts
- ▶  $W$  stabilized, so  $p_k$  doesn't have to change

## Sticky prices model:

differentiated output + homogenous labor

$$p_1 = \frac{W}{A_1}$$

$$p_k = \frac{W}{A_k}$$

With shock to  $A_1$ , want:

- ▶  $p_1$  adjusts
- ▶  $W$  stabilized, so  $p_k$  doesn't have to change

## Standard sticky wage model:

differentiated output + *differentiated* labor

$$p_1 = \frac{W_1}{A_1}$$

$$p_k = \frac{W_k}{A_k}$$

$$W_1 = W_k$$

With shock to  $A_1$ , want:

- ▶  $p_1$  adjusts, so  $W_1 = W_k = p_k$  doesn't have to adjust
- ▶ Wages,  $W_1 = W_k$ , stabilized

- ▶ Suppose  $\psi_P$  if any price  $p_i$  changes
- ▶ Suppose  $\psi_W$  if any wage  $W_i$  changes

## Model:

$$p_1 = \frac{W_1}{A_1}$$

$$p_k = \frac{W_k}{A_k}$$

$$W_1 = W_k$$



- ▶ Suppose  $\psi_P$  if any price  $p_i$  changes
- ▶ Suppose  $\psi_W$  if any wage  $W_i$  changes

## Model:

$$p_1 = \frac{W_1}{A_1}$$

$$p_k = \frac{W_k}{A_k}$$

$$W_1 = W_k$$

**Shock:**  $A_1 \uparrow$

- ▶ Suppose  $\psi_P$  if any price  $p_i$  changes
- ▶ Suppose  $\psi_W$  if any wage  $W_i$  changes

1. **Option 1:**  $p_1$  adjusts

•  $\psi_P$

**Model:**

$$p_1 = \frac{W_1}{A_1}$$

$$p_k = \frac{W_k}{A_k}$$

$$W_1 = W_k$$

**Shock:**  $A_1 \uparrow$

- ▶ Suppose  $\psi_P$  if any price  $p_i$  changes
- ▶ Suppose  $\psi_W$  if any wage  $W_i$  changes

## Model:

$$p_1 = \frac{W_1}{A_1}$$

$$p_k = \frac{W_k}{A_k}$$

$$W_1 = W_k$$

**Shock:**  $A_1 \uparrow$

1. **Option 1:**  $p_1$  adjusts
  - $\psi_P$
2. **Option 2:**  $W_1$  adjusts
  - $\implies W_k$  adjusts  $\implies p_k$  adjusts
  - $(S - 1)\psi_P + S\psi_W$

- ▶ Suppose  $\psi_P$  if any price  $p_i$  changes
- ▶ Suppose  $\psi_W$  if any wage  $W_i$  changes

## Model:

$$p_1 = \frac{W_1}{A_1}$$

$$p_k = \frac{W_k}{A_k}$$

$$W_1 = W_k$$

**Shock:**  $A_1 \uparrow$

1. **Option 1:**  $p_1$  adjusts
  - $\psi_P$
2. **Option 2:**  $W_1$  adjusts
  - $\implies W_k$  adjusts  $\implies p_k$  adjusts
  - $(S-1)\psi_P + S\psi_W$
3. **Option 3:**  $p_k$  adjusts
  - $\implies W_k$  adjusts
  - $(S-1)\psi_W$  and  $W_1 \neq W_k$

- ▶ Suppose  $\psi_P$  if any price  $p_i$  changes
- ▶ Suppose  $\psi_W$  if any wage  $W_i$  changes

## Model:

$$p_1 = \frac{W_1}{A_1}$$

$$p_k = \frac{W_k}{A_k}$$

$$W_1 = W_k$$

**Shock:**  $A_1 \uparrow$

1. **Option 1:**  $p_1$  adjusts
  - $\psi_P$
2. **Option 2:**  $W_1$  adjusts
  - $\implies W_k$  adjusts  $\implies p_k$  adjusts
  - $(S-1)\psi_P + S\psi_W$
3. **Option 3:**  $p_k$  adjusts
  - $\implies W_k$  adjusts
  - $(S-1)\psi_W$  and  $W_1 \neq W_k$

**Optimal policy:**  $p_1$  adjusts,  $W = W_1 = W_k$   
stable

## Optimal policy is not really about selection effects

The existence (or not) of selection effects in menu cost models is an important question in the literature, due to the argument that selection effects reduce monetary non-neutrality relative to models with time-dependent pricing like the Calvo model (Golosov and Lucas 2007; Caballero and Engel 2007; Carvalho and Kryvtsov 2021; Karadi, Schoenle and Wursten 2022). The question this literature generally considers is: in response to a *monetary policy shock*, how much is real output affected? On the other hand, under optimal monetary policy naturally there are no monetary shocks.

However, for the main mechanism we highlight in this paper – a “menu cost channel of optimal monetary policy” – the existence or not of selection effects plays little role. This can be seen by considering two model variants:

1. A menu cost model without selection effects, where firms always set price equal to nominal marginal cost but must pay a menu cost if doing so

# Heterogeneity: a monetary “least-cost avoider principle”

[▶ back](#)

**Proposition 5:** Suppose sector  $i$  has mass  $S_i$  and menu cost  $\psi_i$ . Suppose further

$$S_1\psi_1 < \sum_{k>1} S_k\psi_k.$$

Then optimal policy is exactly as in proposition 1, modulo changes in  $\bar{A}$ .

► *Proof:* Follows exactly as in proof of proposition 1.

# Heterogeneity: a monetary “least-cost avoider principle”

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Then optimal policy is exactly as in proposition 1, modulo changes in  $\bar{A}$ .

► *Proof:* Follows exactly as in proof of proposition 1.

**Interpretation 1: monetary “least-cost avoider principle”**

**Interpretation 2: “stabilizing the stickiest price”**



**Proposition 7:** Consider an arbitrary set of productivity shocks to the baseline model,  $\{A_1, \dots, A_S\}$ .

1. Conditional on sectors  $\Omega \subseteq \{1, \dots, S\}$  adjusting, optimal policy is given by setting  $M = M_\Omega^* \equiv \frac{S-\omega}{\sum_{i \notin \Omega} \frac{1}{A_i}}$ , where  $\omega \equiv |\Omega|$ .
2. The optimal set of sectors that should adjust,  $\Omega^*$ , is given by comparing welfare under the various possibilities for  $\Omega$ , using  $\mathbb{W}_\Omega^*$  defined in the paper.
3. Nominal wage targeting is exactly optimal if the set of sectors which should not adjust are unshocked:  $A_i = 1 \ \forall i \notin \Omega^*$ .

**Proposition 6:** Suppose:

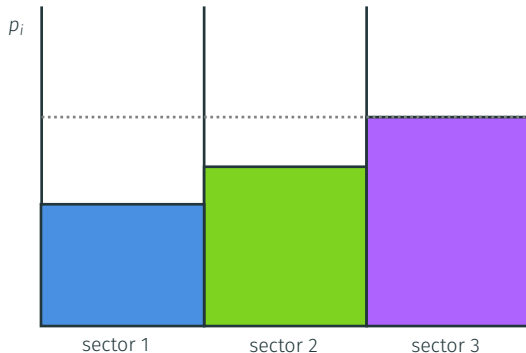
1. Some **strict subset**  $\Omega \subset \{1, \dots, S\}$  of sectors is shocked, with “heterogeneous enough”  $A_i \neq 1$  for all shocked sectors.

Recall:  $p_i^* = MC_i = \frac{W}{A_i}$

**Proposition 6:** Suppose:

1. Some strict subset  $\Omega \subset \{1, \dots, S\}$  of sectors is shocked, with “heterogeneous enough”  $A_i \neq 1$  for all shocked sectors.

Then optimal policy sets  $W = W^{ss}$ .



# Price adjustment frequency tracks inflation in the timeseries

[▶ back](#)

**Calvo/TDP models:** frequency of price adjustment is exogenous to inflation

**Menu cost models:** frequency of price adjustment  $\uparrow$  if inflation  $\uparrow$

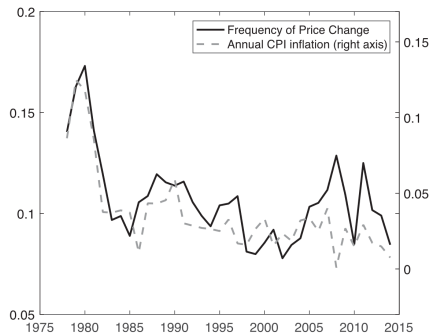


FIGURE XIV

Frequency of Price Change in U.S. Data

**Figure 3:** Nakamura et al (2018)

# Price adjustment frequency tracks inflation in the timeseries

[▶ back](#)

**Calvo/TDP models:** frequency of price adjustment is exogenous to inflation

**Menu cost models:** frequency of price adjustment  $\uparrow$  if inflation  $\uparrow$

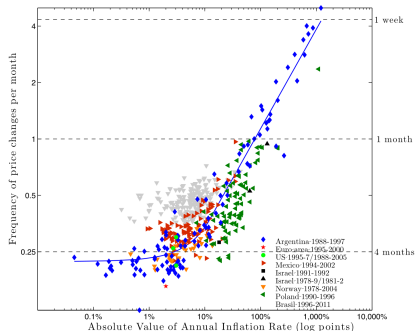


FIGURE VI

The Frequency of Price Changes ( $\lambda$ ) and Expected Inflation: International Evidence

**Figure 3:** Alvarez et al (2018)

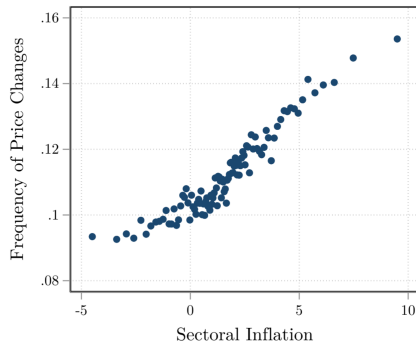
# Price adjustment frequency tracks inflation in the timeseries

[▶ back](#)

**Calvo/TDP models:** frequency of price adjustment is exogenous to inflation

**Menu cost models:** frequency of price adjustment  $\uparrow$  if inflation  $\uparrow$

(a) Frequency of Adjustment



**Figure 3:** Blanco et al (2022)

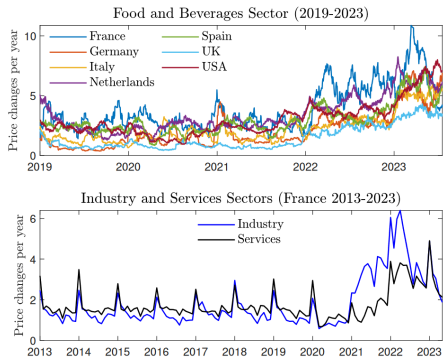
# Price adjustment frequency tracks inflation in the timeseries

[▶ back](#)

**Calvo/TDP models:** frequency of price adjustment is exogenous to inflation

**Menu cost models:** frequency of price adjustment  $\uparrow$  if inflation  $\uparrow$

Figure 1: Frequency of price changes



**Figure 3:** Cavallo et al (2023)

# Evidence of inaction regions

*Figure 8*

**The Distribution of the Size of Price Changes in the United States**

